Automatically estimating noise-induced signal variance in magnitude reconstructed MRI using kernel estimator

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INTRODUCTION

Assessing noise-induced signal variance is important in many MRI applications such as instrumentation quality control, image segmentation, estimation of quantitative parameters from MR data, and functional MRI. Signal variance, σ^2, can be estimated from the noise variance, σ^2_{noise}, in a region of the image containing no object [1, 2]. Extracting the background from the image can be done using automated segmentation. However, the background of MR images, particularly those acquired using echo planar imaging techniques, are rarely free from ghosts and other artifacts. Another approach takes into account that the magnitude MR image measurement can be characterized by a Rayleigh distribution at zero signal to noise [3]. The mean and variance of the noise in the background of magnitude reconstructed images can thus be estimated by fitting the Rayleigh distribution to estimate the probability density function of the data. However, this implementation is susceptible to local optima encountered during the non-linear data fitting procedure. Additionally, because typical MR images have a bimodal or multi-modal distribution containing a mixture of background and foreground values, determining where to fit the distribution to the histogram is not trivial. Another clever approach for estimating the signal variance in MR images is the so called double acquisition method [4]. This approach, however, requires two identical images (with the exception of noise) which are difficult to obtain in practice because of motion that may occur between two different acquisitions. Moreover, artifacts such as ghosting, magnetic susceptibility induced distortions, etc., may not be exactly reproducible in two separate acquisitions. We propose a novel approach for automatically estimating the noise-induced signal variance in magnitude reconstructed MR images, which overcomes shortcomings associated with the methods mentioned above.

METHODS

Let A_r and A_i be the real and imaginary data corrupted by Gaussian distributed noise with zero mean and standard deviation σ. The probability density function (*pdf*) of the magnitude reconstructed data, $A = \sqrt{A_r + A_i}$, is given by a Rician distribution (1) where *M* is the magnitude image intensity and β_0 is the zeroth-order Bessel function of the first kind. At low signal to noise (S/N), this distribution approaches the Rayleigh distribution (2) with mean $\overline{M} = \sigma\sqrt{\pi/2}$ and variance $\sigma^2_{noise} = (2-\pi/2)\sigma^2$. At high S/N, the noise distribution approaches a Gaussian distribution (3) with mean $\overline{M} = \sqrt{A^2+\sigma^2}$ and variance $\sigma^2_{noise} = \sigma^2$.

$$p(M) = \frac{M}{\sigma^2} e^{-(M^2+A^2)/(2\sigma^2)} \beta_0\left(\frac{A\cdot M}{\sigma^2}\right) \quad (1)$$

$$p(M) = \frac{M}{\sigma^2} e^{-M^2/(2\sigma^2)} \quad (2)$$

$$p(M) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(M-\sqrt{A^2+\sigma^2})^2/(2\sigma^2)} \quad (3)$$

$$\frac{\partial p(M)}{\partial M} = \frac{1}{\sigma^2} e^{-M^2/(2\sigma^2)} + \frac{M}{\sigma^2}\left(\frac{-2M}{2\sigma^2}\right) e^{-M^2/(2\sigma^2)} = 0 \quad (4)$$

Note that the value of *M* for which *p*(*M*) in (2) is maximum is equal to σ. This finding can be easily checked by differentiating (2) with respect to *M*, setting the result equal to zero, and solving for *M* (4). This information can be used to extract the standard deviation of the signal intensity σ in the images by simply identifying the peak of the noise distribution. We use the kernel or the Parzen density estimator [5] for this purpose, the most popular technique for nonparametric density estimation. The choice of basis function is not very important so long as it is smooth and bell-shaped [5]. We chose the Gaussian kernel because it is easy to manipulate and derive. The kernel size or window width is very important and sometimes is adapted to the application. There is a trade-off between too much variability on one hand (if the window width is too small) and increased bias on the other (if it is too large). The window width can be computed by minimizing the mean square error between the true and estimated density. In our simulation, we set the window width equal 1.06*(sample standard deviation)*$n^{-1/5}$ where n is the sample size as suggested in [5]. We created synthetic images containing different size objects and added Gaussian distributed noise in quadrature to simulate images with different S/N. Our objective is to test the accuracy of the proposed method.

RESULTS

Figure 1 shows the simulation result on the amount of background required for the proposed method to properly estimate the peak of the noise distribution. If the error of the estimated signal standard deviation is set within 10%, 65% of the background in an image is required when S/N =3, 22% is required when S/N=4, and only a small amount of background is needed when S/N ≥ 5. In general, more background would provide a better result. If the background is less than required, for example, less than 60% when S/N=3, the estimated signal standard deviation is somewhat over-estimated. This is understandable because the noise distribution is contaminated by the "object signal" and the mixture of noise and object signal will always cause the estimated peak to be shift to the right.

We use echo-planar diffusion weighted images as a test application. We carefully chose 30 regions manually from the background and compute the average of variance from those regions. Our preliminary result shows that the estimated signal variance using the Parzen window approach has similar results when compared with this conventional estimation (has the error rate within 10% in our testing data sets). Simulations with artifacts will be conducted to further verify our preliminary findings.

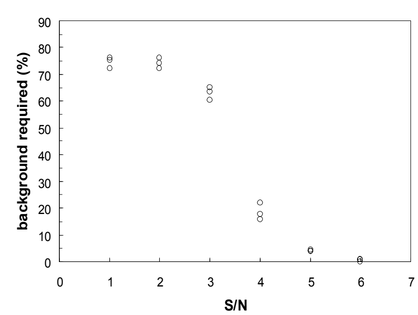

Fig. 1 The background required for the kernel estimator to properly estimate the signal standard deviation (0.9σ < $\sigma_{estimated}$ < 1.1σ) from an image (512*512).

CONCLUSION

An automatic method for estimating the signal variance in magnitude reconstructed MRI is presented. This method needs only one image, does not require any user interaction as no background pixels need to be selected, and does not require prior brain segmentation. The result is promising when compared with the conventional manual object-free background selection.

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